Mechanical Simulation Analysis of Reciprocating Pump Crosshead with Clearances in Prismatic Pair

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Abstract: Mechanical and mathematical model of reciprocating pump crosshead with clearance in prismatic pair was eastablished. Using the numerical solution method of Runge-Kutta, the mechanical simulation calculation of crosshead is completed by computer. It also analyses the influence of offset rate, clearance value and crankshaft speed on the impact force of crosshead. The results show that, the reciprocating pump is forward rotating, the positive pressure between the crosshead and the guide rail can be obviously improved by the reasonable positive bias of the crosshead. When the clearance of prismatic pair is included, the improvement effect of the positive bias of the crosshead on the maximum impact force of the crosshead is limited, and the influence of the clearance of prismatic pair on the mechanical properties of the crosshead cannot be ignored.

1. Introduction

After the reciprocating pump runs for a long time, the clearance will appear due to friction and wear between kinematic pair. The clearance between the crosshead and the crosshead guide is very small but large enough to induce the crosshead secondary motions (crosshead motions in both Vertical and rotational directions are called crosshead secondary motions) and generates unwanted vibration and sound [1]. This secondary motion is caused by the connecting rod force that changes its direction depending on its position and the crosshead moves from one side to the opposite side in the machine (the crosshead collides against the inner surface of the crosshead guide) [1].

In the past few decades, scholars have done a lot of research on revolute clearance joint, spherical clearance joint and column clearance pair, but few on prismatic clearance pair. There have been many attempts to model or estimate the impact forces and the connecting rod force in internal combustion engine [2, 3]. Some models are also established to study the dynamic forces of crossheads that cause vibration in reciprocating compressors [1]. There are also studies on the dynamics forces of translation joints with clearance in rigid multibody systems [4-6]. In contrast to the internal combustion engine and reciprocating compressors, it is lack of work to model the prismatic pair with clearance in reciprocating pump.

We need, therefore, to study the mechanical characteristics of reciprocating pump crosshead with clearance in prismatic pair. This paper describes how we establish the mechanical and mathematical model of the crosshead with clearance in prismatic pair. On this basis, the impact forces of crosshead skirt is estimated, and the influence of the bias ratio of the crosshead, the clearance value and the crankshaft speed on the impact force of the crosshead skirt is analyzed.

2. Simplified Model

In this paper, the crankshaft, connecting rod and crosshead in reciprocating pump are simplified as plane crank slider mechanism. The simplified model of crank slider mechanism with clearance of slding pair is shown in figure1.

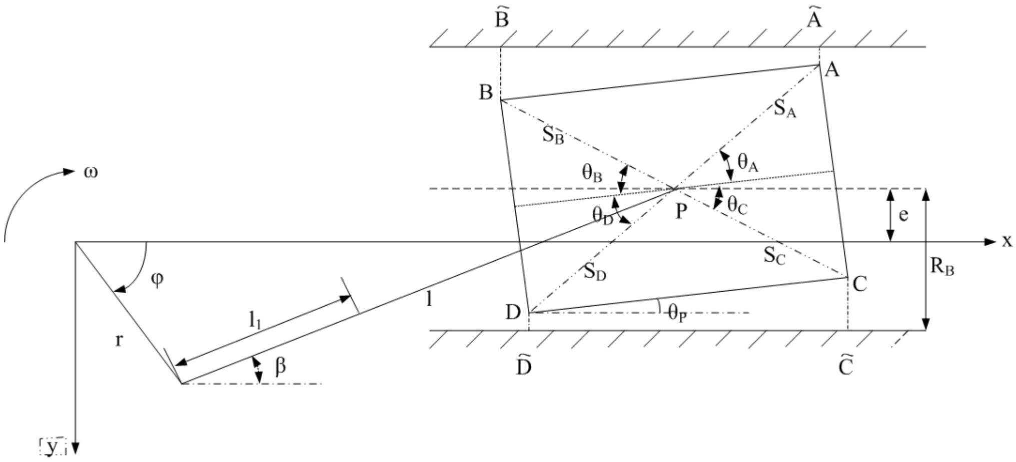


Figure 1. Simplified model of plane crank slider mechanism

In contrast to the ideal prismatic pair, the existence of a clearance in a prismatic pair removes two kinematic constraints from a planar system and introduces two extra degrees of freedom in the system. In a prismatic pair with clearance, there are four contact forms between the crosshead and guide, as shown in figure 2, namely: (a) two adjacent crosshead corners in contact with the guide surface, (b) one corner of the crosshead in contact with the guide surface, (c) two opposite crosshead corners in contact with the guide surfaces, (d) no contact between the two elements.

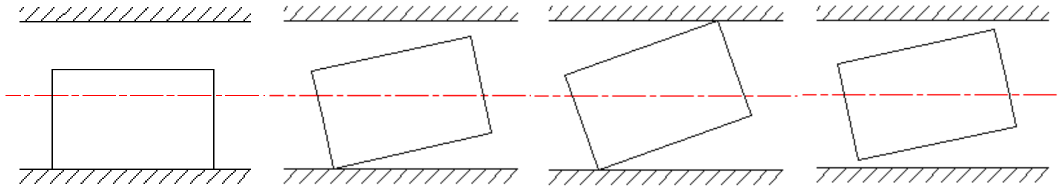


Figure 2. Different contact forms between the crosshead and guide

In order to complete the precise mechanics simulation analysis of crosshead with prismatic clearance pair, on the basis of establishing the clearance model of the prismatic pair in line with the reality, we should apply certain mathematical description method to realize the continuity of the dynamic equation of the mechanism with clearance, and accurately judge the change of contact forms between crosshead and guide in the contact collision process. According to the rectangular coordinate system established in figure 1, referring to the geometric position of the collision point of the crosshead skirt and the collision point of the guide, the intrusion amount of each collision point position can be obtained as follows:

$$\delta_A = \tilde{y}_A - y_A, \quad \delta_B = \tilde{y}_B - y_B, \quad \delta_C = y_C - \tilde{y}_C, \quad \delta_D = y_D - \tilde{y}_D \tag{1}$$

The contact collision discrimination condition is introduced to describe the state of each collision point of the crosshead, and the contact collision discrimination condition is shown in Eq. (2).

$$\begin{cases} \delta_i < 0, & \text{free separation} \\ \delta_i = 0, & \text{noncontact collision} \\ \delta_i > 0, & \text{contact collision} \end{cases} \tag{2}$$

3. Mechanical Model

In the process of reciprocating movemnet, crosshead is mainly subjected to connecting rod force, contact impact force, friction force, gravity and plunger force, as shown in the figure 3.

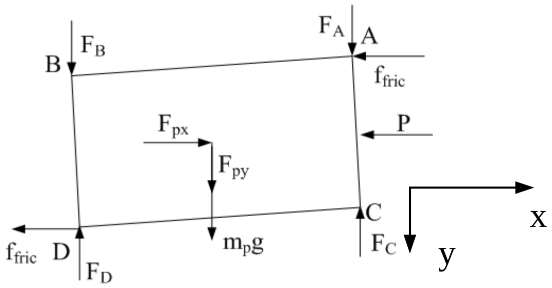


Figure 3. Forces on the crosshead

The plunger force is expressed as

$$P = \begin{cases} -pA, & \dot{x}_p < 0 \\ 0, & \dot{x}_p > 0 \end{cases} \tag{3}$$

Where p and $\dot{x}_p$ are liquid pressure and velocity of crosshead. A is the cross-section area of the plunger, $A=\pi d^2/4$, parameter d is diameter of plunger.

Considering the existence of the clearance between the crosshead and the guide, the existence of coulomb friction force between the crosshead and the guide rail is taken into account temporarily under the condition of small deformation, and the tangential friction force of the crosshead under the condition of contact collision is not taken into account to solve the connecting rod force.

Connecting rod force and the direction component of its in the coordinate axis are

$$F_p = \frac{m_p \ddot{x}_p - P + u m_p g}{\cos\beta - u \sin\beta} \tag{4}$$

$$F_{px} = F_p \cos\beta \tag{5}$$

$$F_{py} = F_p \sin\beta \tag{6}$$

where m_p and u are the crosshead mass and coefficient of coulomb frication.

The contact force is mainly composed of normal collision force and tangential friction force in the contact area between the crosshead and the guide. As for tangential friction force, this paper adopted Coulomb friction model to describe the friction behavior in the collision process between crosshead and guidel, without considering the oil film pressure generated by the oil film. Tangential friction is expressed as

$$f_{fric}^{i} = -\mu_d F_i \operatorname{sgn}(v), \quad \text{i=A, B, C, D} \tag{7}$$

where μ_d , F_i and sgn(v) are the coefficient of sliding friction, the impact force at each impact point of the crosshead and sign function, The sign function is given by

$$\operatorname{sgn}(\dot{x}_p) = \begin{cases} 1 , & \dot{x}_p > 0 \\ -1, & \dot{x}_p < 0 \end{cases} \tag{8}$$

The formula of normal contact impact force is

$$F_i = K\delta_i^n + D\dot{\delta}_i \quad \text{n=1.5, i=A, B, C, D} \tag{9}$$

where K, D and δ_i are the stiffness coefficient, damping coefficient and relative impact velocity.

The stiffness coefficient can be expressed as

$$K = \frac{4\sqrt{R_i}}{3\pi(\sigma_i + \sigma_j)} \tag{10}$$

where R_i, σ_i and σ_j are the radius of curvature of an imaginary sphere, material coefficient of crosshead and guide. The material coefficient of corsshead and guide are given by

$$\sigma_K = \frac{1-\mu_k^2}{E_k}, (k = i, j) \tag{11}$$

where μ_k and E_k are the poisson ratios and elasticity modulus.

The damping coefficient can be expressed as

$$D = \mu\delta^n \tag{12}$$

where μ is the hysteretic damping coefficients, it can be given by

$$\mu = \frac{3K(1-e^2)}{4\dot{\delta}^{(-)}} \tag{13}$$

parameters e and $\dot{\delta}^{(-)}$ are the coefficient of restitution and initial relative impact velocity.

Eq.(9) is applicable to a vertex or two oppositive vertices of the crosshead in contact with the guide rail surface, and when the crosshead is in contact with the guide rail surface or free separation state, we define F_i as 0.

In a word, the normal contact collision force is

$$F_i = \begin{cases} K\delta_i^n + D\dot{\delta}_i \ , & \delta_i > 0 \\ 0 \ , & \delta_i \le 0 \end{cases} \tag{14}$$

4. Mathematical Model

In the vertical and rotational directions, dynamic equations of the crosshead can be written as Eq. (15).

$$\begin{cases} m_p \ddot{y}_p = F_A + F_B - F_C - F_D + F_{py} + m_p g \\ I_p \ddot{\theta}_p = -P\cdot(-e - x_p) + F_A \cdot l_{xA} - F_B \cdot l_{xB} - F_C \cdot l_{xC} + F_D \cdot l_{xD} \\ \qquad + f_{fric}^A \cdot l_{yA} + f_{fric}^B \cdot l_{yB} - f_{fric}^C \cdot l_{yC} - f_{fric}^D \cdot l_{yD} \end{cases} \tag{15}$$

Eq. (15)'s the system of first-order ordinary differential equations are rewritten as Eq. (16).

$$\begin{cases} \frac{d\dot{y}_p}{dt} = (F_A + F_B - F_C - F_D + F_{py} + m_p g) / m_p \\ \frac{dy_p}{dt} = \dot{y}_p \\ \frac{d\dot{\theta}_p}{dt} = (-P\cdot(-e - x_p) + F_A \cdot l_{xA} - F_B \cdot l_{xB} - F_C \cdot l_{xC} + F_D \cdot l_{xD} \\ \qquad + f_{fric}^A \cdot l_{yA} + f_{fric}^B \cdot l_{yB} - f_{fric}^C \cdot l_{yC} - f_{fric}^D \cdot l_{yD}) / I_P \\ \frac{d\theta_p}{dt} = \dot{\theta}_p \end{cases} \tag{16}$$

where l_{xi} and l_{yi} are the x-axis and y-axis component of the distance from each impact points to the center of mass of the crosshead.

5. Calculation Results and Analysis

(1) Calculation of the positive pressure of crosshead without clearance

The formula (3-33) was used to calculate the positive pressure, see reference [7]. The simulation curves of positive pressure under positive offset of crosshead and positive rotation of reciprocating pump was shown as figure 4. The maximum positive pressure value and improvement rate of crosshead under different offset ratios was shown as table 1. Under different connecting rod ratios, the calculation of the optimum offset ratio of the crosshead are also calculated, see table 2.

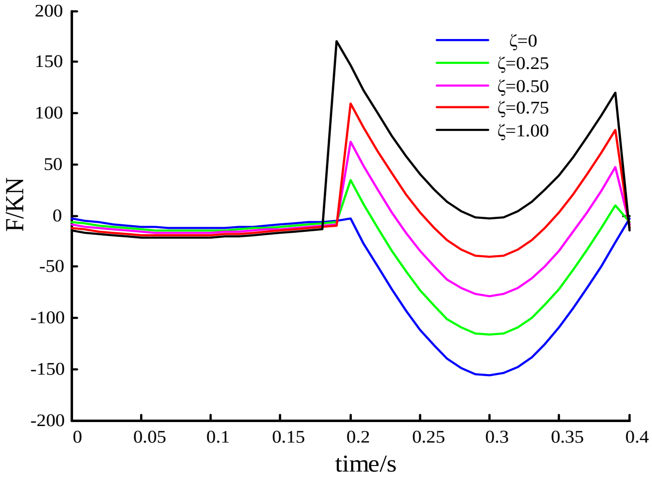


Figure 4. The positive pressure of the crosshead

Table.1. The maximum positive pressure value and improvement rate

Offset ratio	Maximum positive pressure/KN	Improvement rate
0	156.076	0
0.25	116.846	25.14 %
0.5	78.387	49.78 %
0.75	108.805	30.29 %
1	169.921	-8.87 %

Table.2. The optimum offset ratio of the crosshead

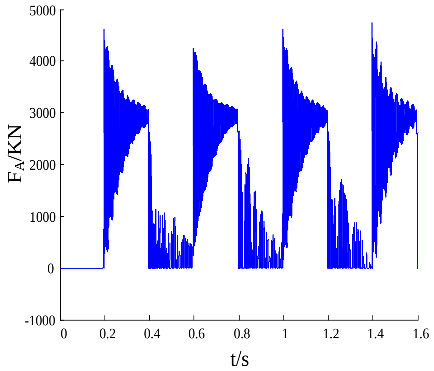
λ	**ζ**	**δ**
0.05	0.6	53.9 %
0.1	0.53	50.7 %
0.1395	0.5	49.78 %
0.15	0.5	49 %
0.2	0.475	47.7 %
0.25	0.458	46.5 %
0.3	0.435	45.5 %

(2) Calculation of the impact forces of crosshead with clearance

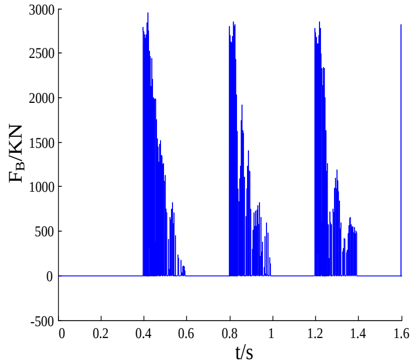
Runge-Kutta method is adopted to solve the mathematical model by using MATLAB programming, the basic parameters as shown in table 3. Then the impact forces on the skirt of the crosshead with prismatic pair clearance is simulated, see figure5.

Table.3. The basic parameters

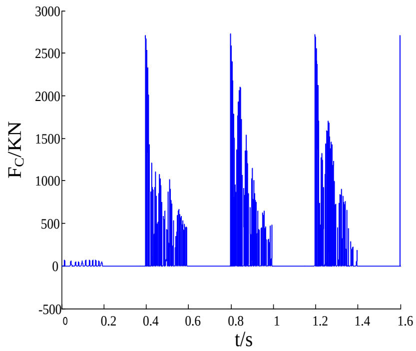
Parameters	**Numerical size**	**Unit**
r	120	mm
l	860	mm
e	60	mm
d	101.6	mm
p	140	Mpa
n	150	r/min
c	1	mm



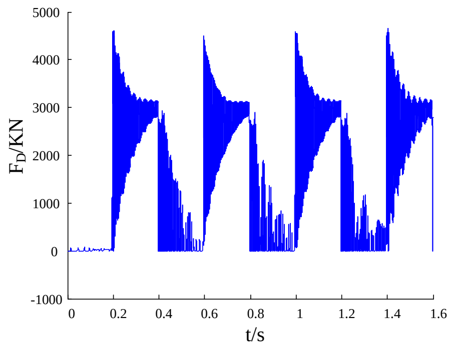
(a) impact force of point A



(b) impact force of point B



(c) impact force of point C



(d) impact force of point D

Figure 5.The impact forces on the skirt of the crosshead

Under the conditions of positive offset and centering type of the crosshead, the maximum skirt impact forces of crosshead with clearance in prismatic pair is shown in figure 6. The forces between the crosshead and the guide under three conditions of centering type without prismatic pair clearance, centering type with prismatic pair clearance and positive bias with prismatic pair clearance are shown in table 4.

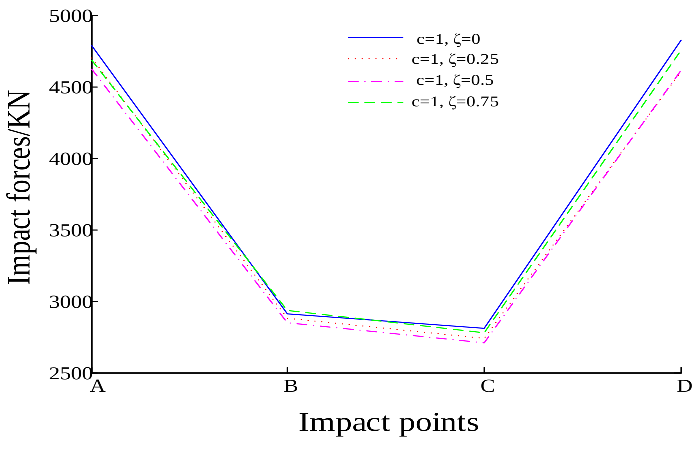


Figure 6. The maximum skirt impact forces

Table.4. The impact forces on three conditions

Different conditons	**F_A/KN**	**F_B/KN**	**F_C/KN**	**F_D/KN**	**F/KN**
ζ=0, c=0	--	--	--	--	156.08
ζ=0, c=1	4787.48	2911.61	2805.33	4827.41	--
ζ=0.5, c=1	4618.49	2850.14	2707.36	4610.66	--

The changes of the maximum skirt impact forces of the crosshead with different clearance values are shown in figure 7, and the changes of the maximum skirt impact forces of the crosshead with different crankshaft speeds are shown in figure 8.

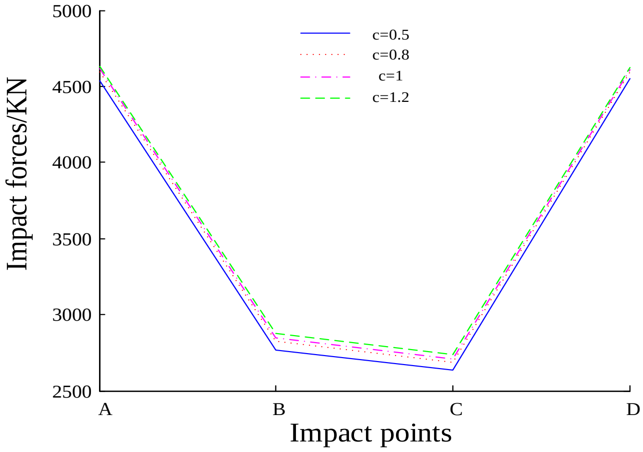


Figure 7. The maximum skirt impact forces

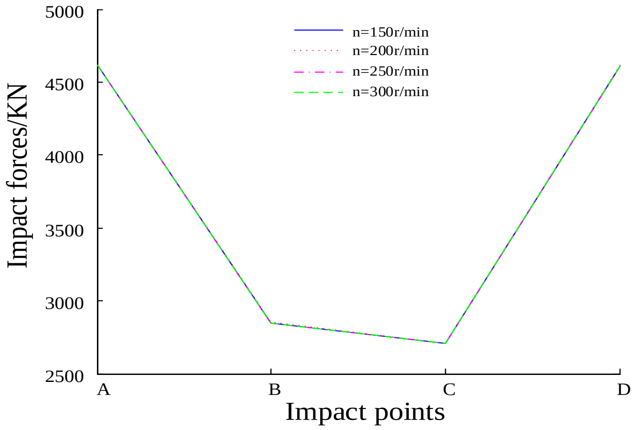


Figure 8. The maximum skirt impact forces

6. Conclusion

Through the analysis for the simulation results in this paper, we can get the conclusion as follows:

(1) The positive pressure between the crosshead and the guide can be significantly reduced by the rational positive offset of the cross head, and the optimal deviation ratio is different for different connecting rod ratios. For example, in one certain type reciprocating pump, λ=0.1395, the maximum positive pressure between the crosshead and guide is reduced by 49.78%.

(2) In the case of prismatic pair with clearance, the improvement effect of crosshead positive bias on maximum impact forces of skirt is 3% to7%.

(3) The impact forces increase as the clearance size is increased. The increase of crankshaft speed has little effect on the change of impact forces on the skirt of the crosshead at the most. However, as the running period of the crosshead decreases, the collision times between the crosshead and the guide in unit time increase significantly.

The influence of prismatic pair clearance on the mechanical properties of the crosshead can not be ignored.

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References

[1] A.Almasi. A new study and model for the mechanism of process reciprocating compressors and pumps.Process of mechanical engineering, 224 (2010), pp. 143-147.

[2] S.H. Cho, S.T. Ahny, H. Kim. A simple model to estimate the impact force induced by piston slap. Journal of sound and vibration, 255(2002), pp. 229-242.

[3] Y.C. Tan, Z.M. Ripin. Analysis of piston secondary motion [J]. Journal of sound and vibration, 332(2013), pp. 5162-5176.

[4] O.A. Bauchau, J. Rodriguez, Modelling of joints with clearance in flexible multibody systems. Int. J. Solids Struct. 39(2002), pp. 41–63.

[5] P. Flores, H. M. Lankarani, J. Ambrósio, J. C. P. Claro. Modeling lubricated revolute joints in multibody mechanical systems. Proc. Inst. Mech. Eng., Part K: J. Multi-body Dynamics, 218 (2004), pp. 183–190.

[6] P. Flores, J. Ambrosio, J. C. P. Claro, H. M. Lankarani. Translational joints with clearance in rigid multibody systems. Journal of computational and nonlinear dynamics, 3 (2008), pp. 1-10.

[7] X.H. Shen. Principle and design of drilling reciprocating pump. China machine press, Beijing, 1990, pp. 52.